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Why stars explode

Outline

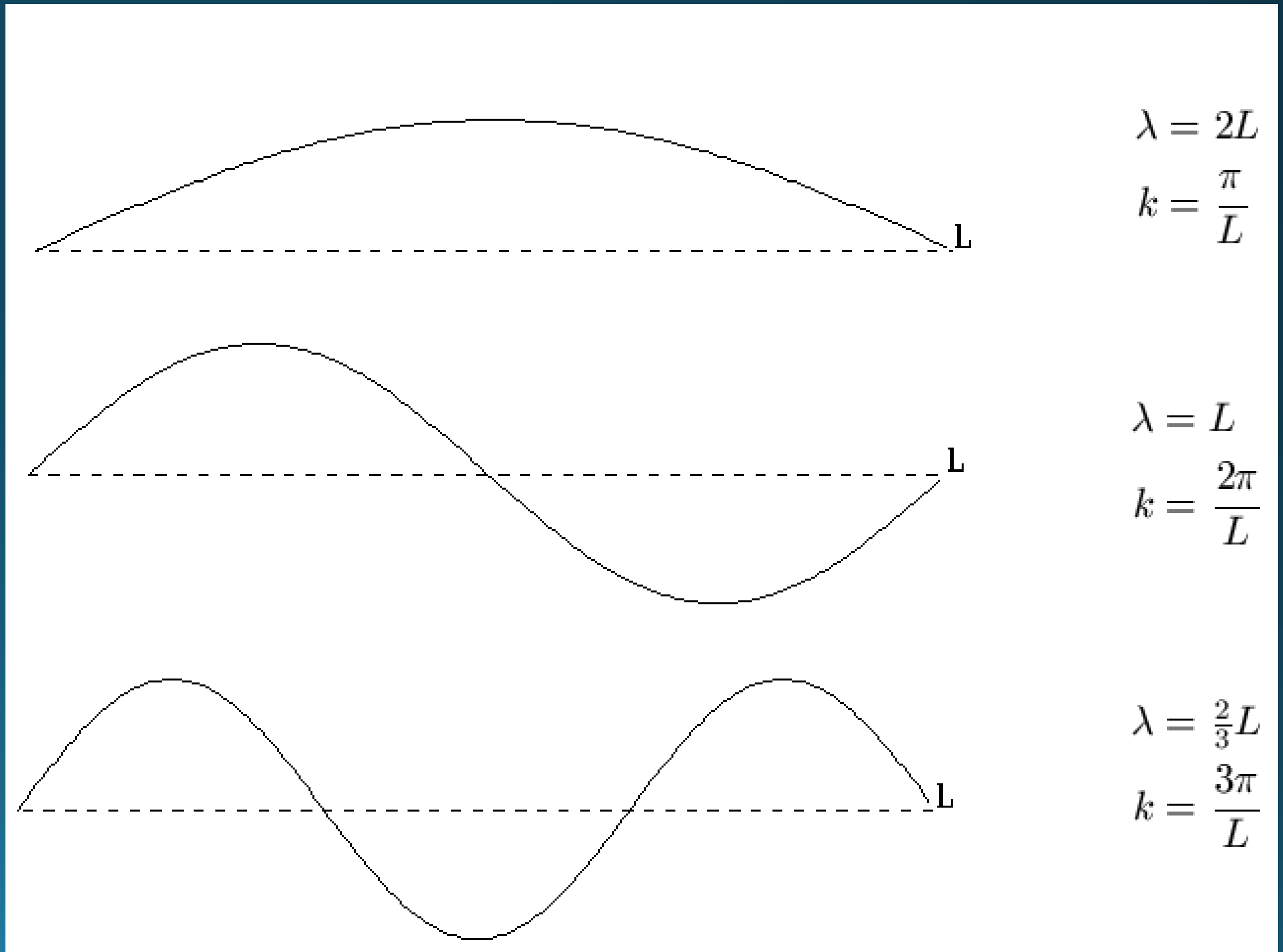
- How stars age
- The physics of a degenerate gas
- Application to stars
- 2 types of commital
  - burial and cremation

How stars age

- Stars radiate energy
- Self-gravitating objects have negative specific heat (Sat Jan 2018)
  - So they respond to heat loss by heating up
- They heat by contracting/compressing
  - Gravity now stronger so more pressure needed
- Moderate rise in T greatly enhances rates of nuclear reactions
  - They are driven by collisions of nuclei in the high-E tail of the Maxwellian
- As charge on nuclei rises, higher T required to drive fusion
- Summary
  - Through stellar life central T rises while ρ rises more strongly
- Conclusion
  - Cores degenerate in old age

Gas in quantum mechanics

- Classical kinetic theory not valid at high density
  - → QM needed
- Pressure mostly provided by Fermions *e* or n
- Pauli exclusion principle:
  - max 1 particle in each state
- So count states
  - Box L on a side
  - k= n(π/L)



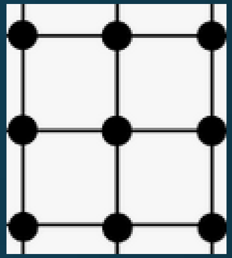
State counting

In 3d $\mathbf{k} = (k_x, k_y, k_z)$ with $k_i = n_i \frac{\pi}{L}$
$\mathbf{k}$ is position vector of points on a lattice spaced by $\Delta k = \frac{\pi}{L}$
Number of points in sphere radius k_{F} is

$$\frac{\frac{4}{3}\pi k_{\mathrm{F}}^3}{(\Delta k)^3} = \frac{4}{3\pi^2}(k_{\mathrm{F}} L)^3$$

For spin $\frac{1}{2}$ particle, 2 states for each point.
Also $p = \hbar k$, so number of states with $p \leq p_{\mathrm{F}}$

$$N = \frac{8}{3\pi^2}\left(\frac{p_{\mathrm{F}} L}{\hbar}\right)^3$$



Enter relativity

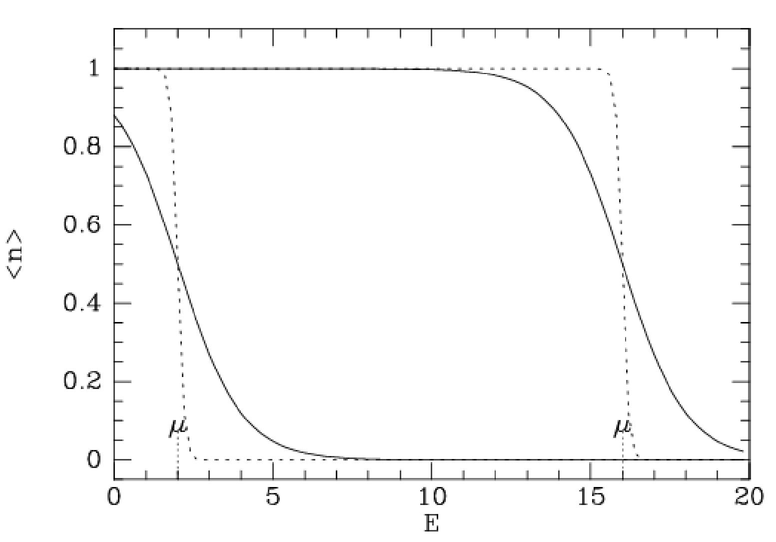
If we have N particles in box, *minimum* momentum of fastest particles is *Fermi momentum* p_{F}
Particle density $n = N/L^3 = \frac{8}{3\pi^2}(p_{\mathrm{F}}/\hbar)^3 \Rightarrow$

$$p_{\mathrm{F}} = K v^{-1/3} \text{ with } \begin{cases} K \equiv (3\pi^2/8)^{1/3}\hbar \\ v \equiv 1/n \end{cases}$$

We'll find that stellar cores have $kT \ll E(p_{\mathrm{F}})$ so states up to p_{F} full, states above empty
Special relativity:

$$m^2c^2 = \frac{E^2}{c^2} - p^2$$

$$\Rightarrow \quad \frac{E_{\mathrm{F}}\,\mathrm{d}E_{\mathrm{F}}}{c^2} = p_{\mathrm{F}}\,\mathrm{d}p_{\mathrm{F}} = -p_{\mathrm{F}}\,\tfrac{1}{3}Kv^{-4/3}\mathrm{d}v = -\tfrac{1}{3}K^2v^{-5/3}\mathrm{d}v$$



Adiabatic compression

2nd law of thermodynamics:

$$\mathrm{d}U = T\,\mathrm{d}S - PN\,\mathrm{d}v \quad \text{with} \quad U = xNE_\mathrm{F}$$

with $\frac{3}{5} \leq x < \frac{3}{4}$, so

$$P = -x\frac{\partial E_\mathrm{F}}{\partial v} = A\frac{v^{-5/3}}{E_\mathrm{F}} \quad \text{where} \quad A = \tfrac{1}{3}K^2c^2x$$

with $\partial E_\mathrm{F}/\partial v$ from last slide

Two adiabatic laws

$$p_{\mathrm{F}} = K v^{-1/3} \qquad P = A\frac{v^{-5/3}}{E_{\mathrm{F}}}$$

$$m^2c^2 = \frac{E^2}{c^2} - p^2$$

If non-relativistic $E_{\mathrm{F}} \simeq mc^2$ so $Pv^{5/3} = \text{constant}$

If ultra-relativistic $E_{\mathrm{F}} \simeq cp_{\mathrm{F}}$ so $Pv^{4/3} = \text{constant}$

In either case $Pv^{\gamma} = \text{constant}$

Enter gravity

Ball of gas radius R, so $\mathrm{d}v = 4\pi R^2/N\,\mathrm{d}R$

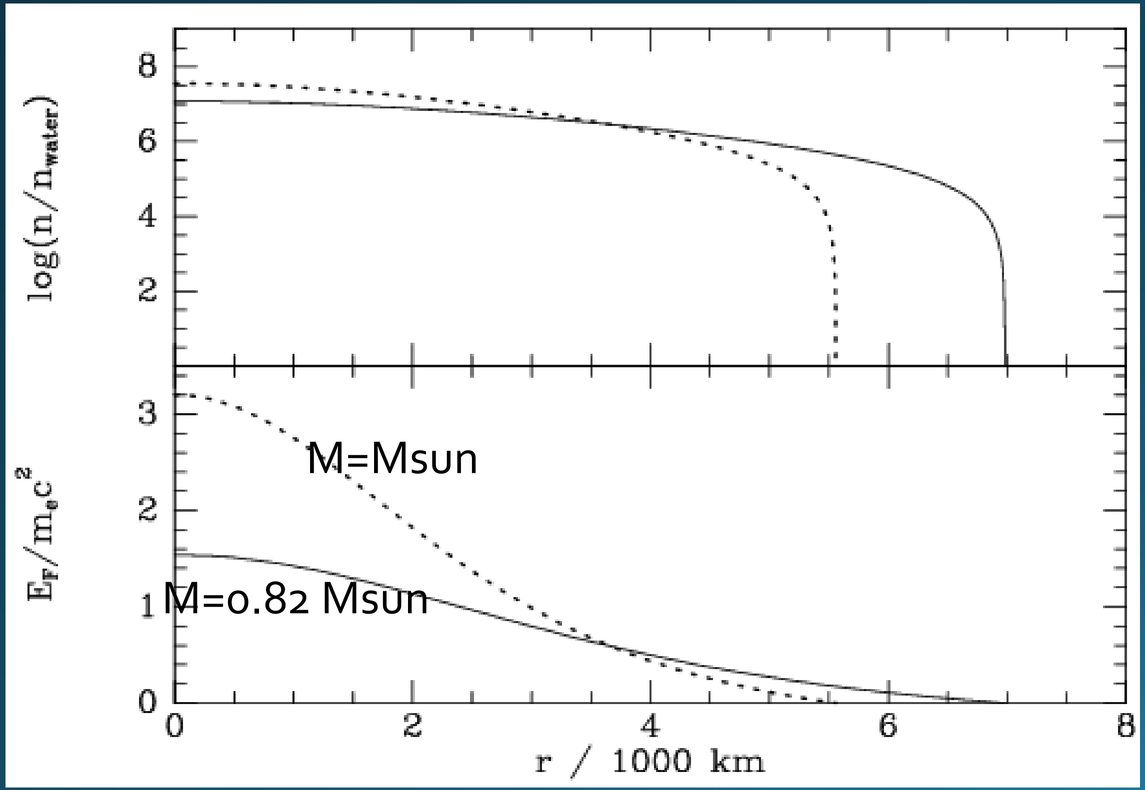
$$\begin{aligned}\mathrm{d}U &= -PN\mathrm{d}v = -\mathrm{const}\times v^{-\gamma}\mathrm{d}v \\ &= \mathrm{const}\times R^{-3\gamma+2}\,\mathrm{d}R = \mathrm{const}\times\begin{cases} R^{-3}\,\mathrm{d}R \text{ non-rel} \\ R^{-2}\,\mathrm{d}R \text{ ultra-rel}\end{cases}\end{aligned}$$

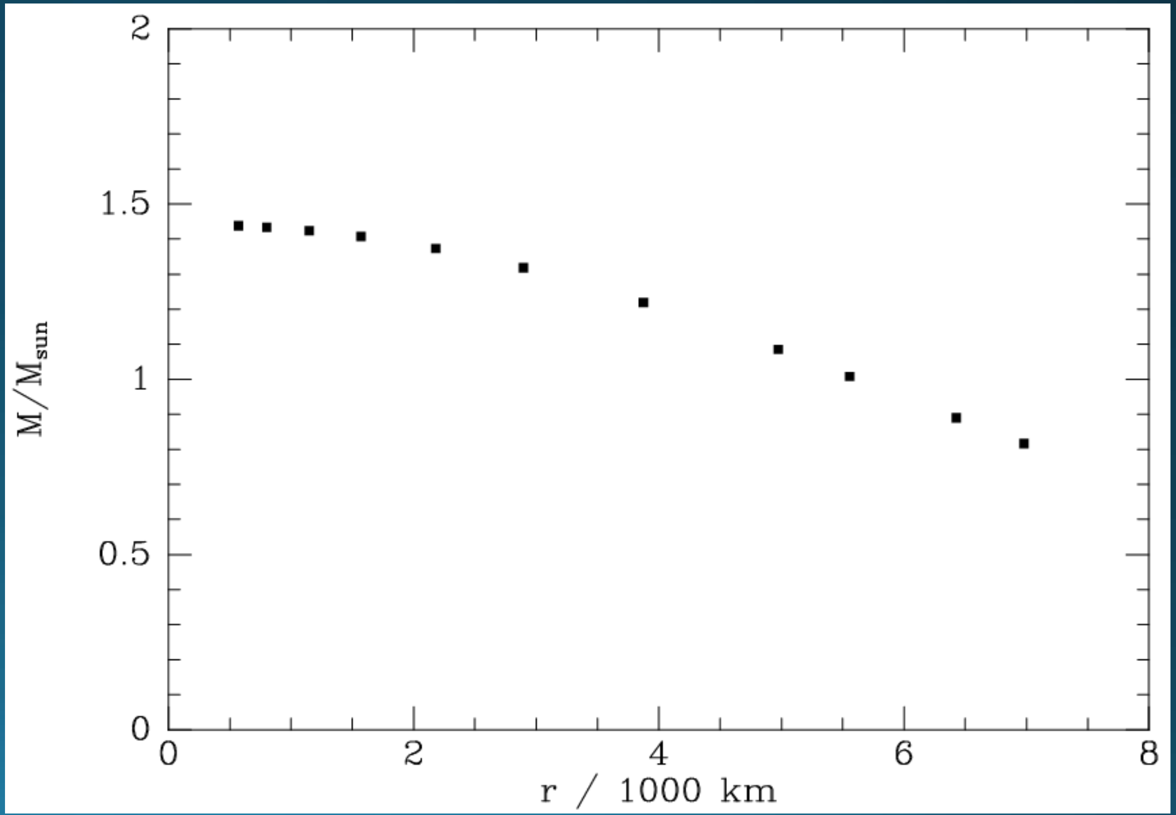
$$U_{\mathrm{grav}} = -y\frac{GM^2}{R} \quad\Rightarrow\quad \mathrm{d}U_{\mathrm{grav}} = y\frac{GM^2}{R^2}\mathrm{d}R \text{ with } y\sim 1$$

Non-rel case: on contraction U rises faster than U_{grav} diminishes: stable
Ultra-rel case: U rises at same rate as U_{grav} drops so neutral stability
GR tips the balance towards contraction because P as well as M generates gravity

Numerical results

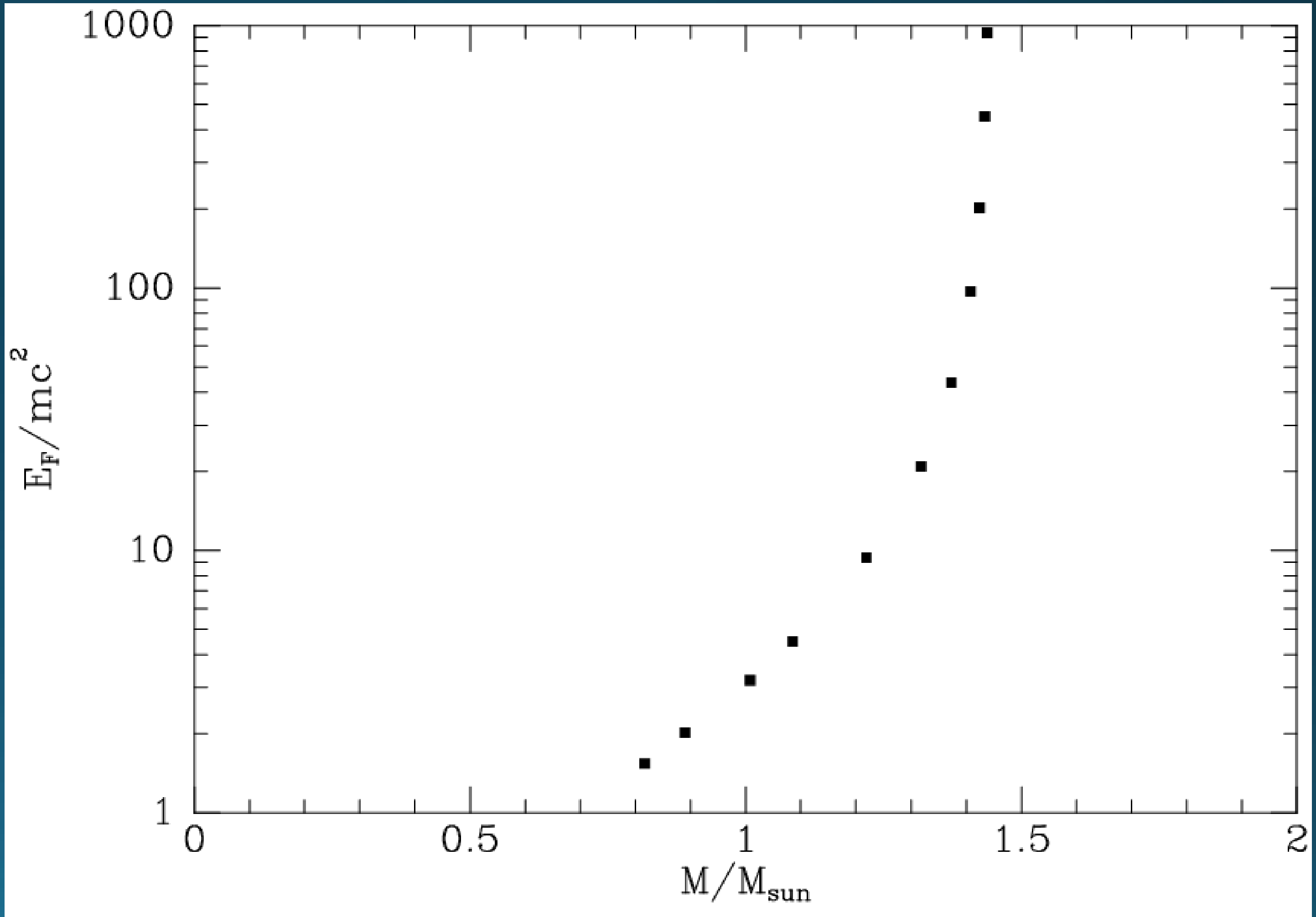
- Radius of WD *decreases* as M increases
  - R→0 as M→1.44Msun (Chandrasekhar) [$M_C \sim (hc/Gm_n^2)^{3/2}$]
- Electrons become relativistic at M ~ 0.5 Msun





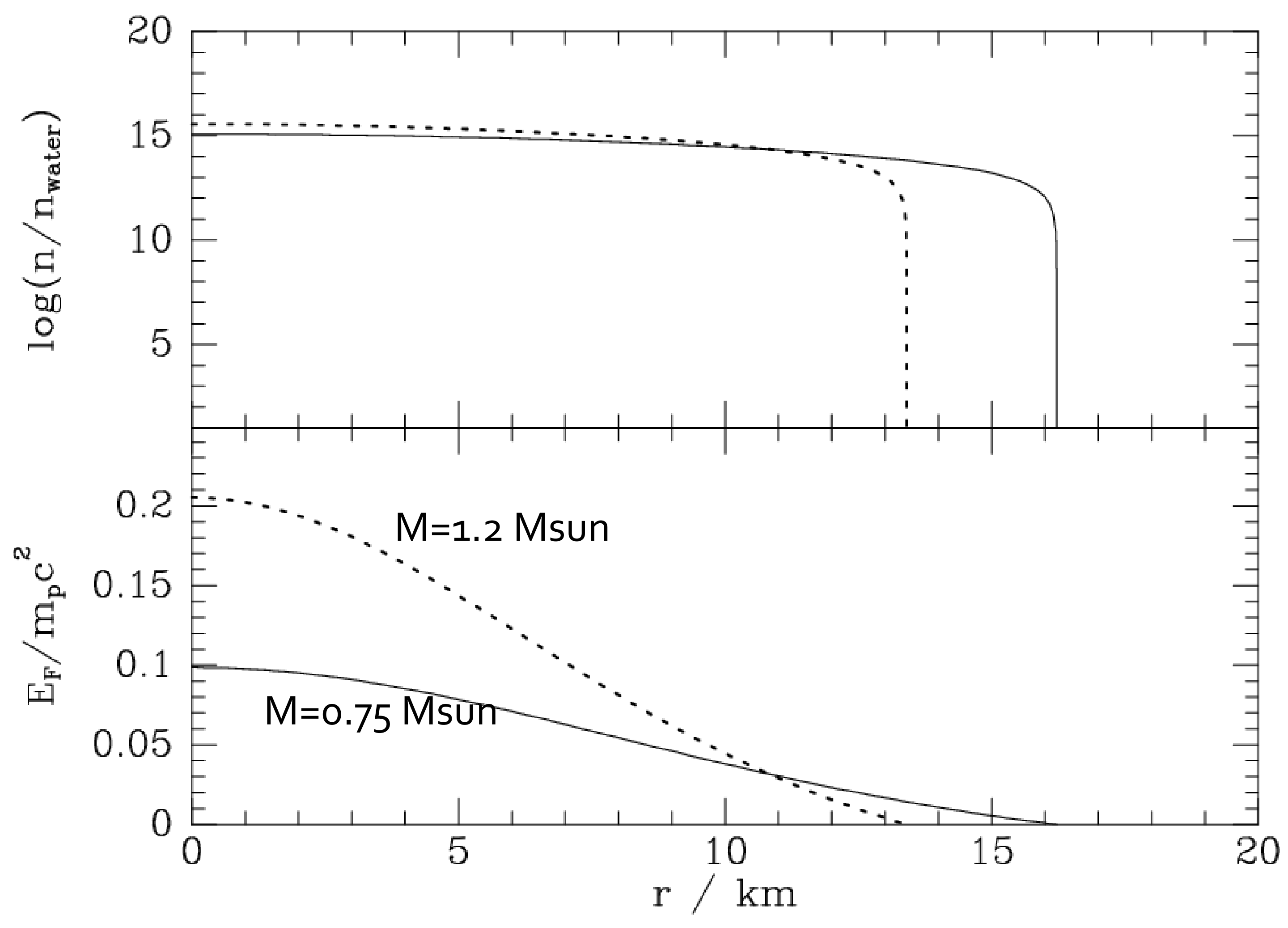
Central $E_F(M)$

- E_F shoots up as
- $M \rightarrow M_{Chandra}$



Neutron-star models

- Fundamentally same as WD model
- Changes to r and n scales but not to M scale
- Caveat
  - NSs not ideal gas

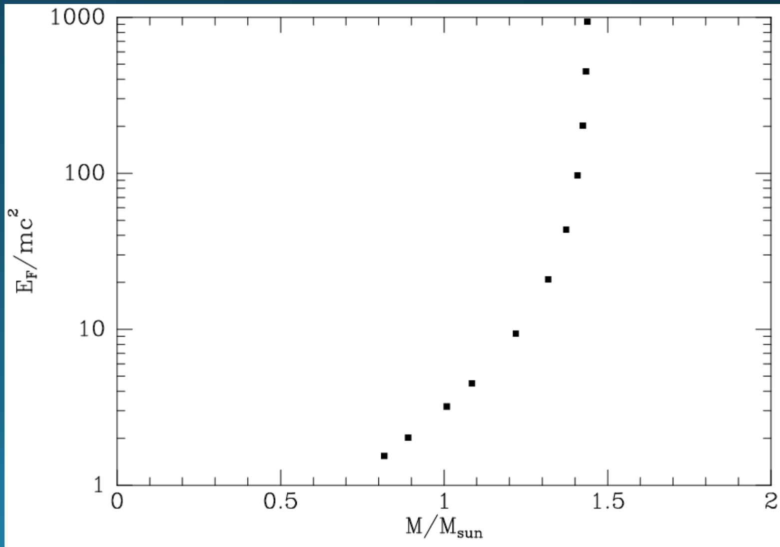


Stellar evolution

- Stars with $0.5M_{sun} < M_i < 8\ M_{sun}$ turn H → He → C,O
- Then envelope blown away as a planetary nebula
- Leaves white dwarf: no bigger than Earth, $M \sim M_{sun}$, supported by E_F of electrons
- Set to cool for ever
- Stars with $M_i > 8M_{sun}$ turn H → He → C,O → Si → Fe
- → Fe core $M \sim M_{sun}$ supported by E_F of electrons
  - Essentially an Fe WD
- Either endpoint can give rise to an explosion

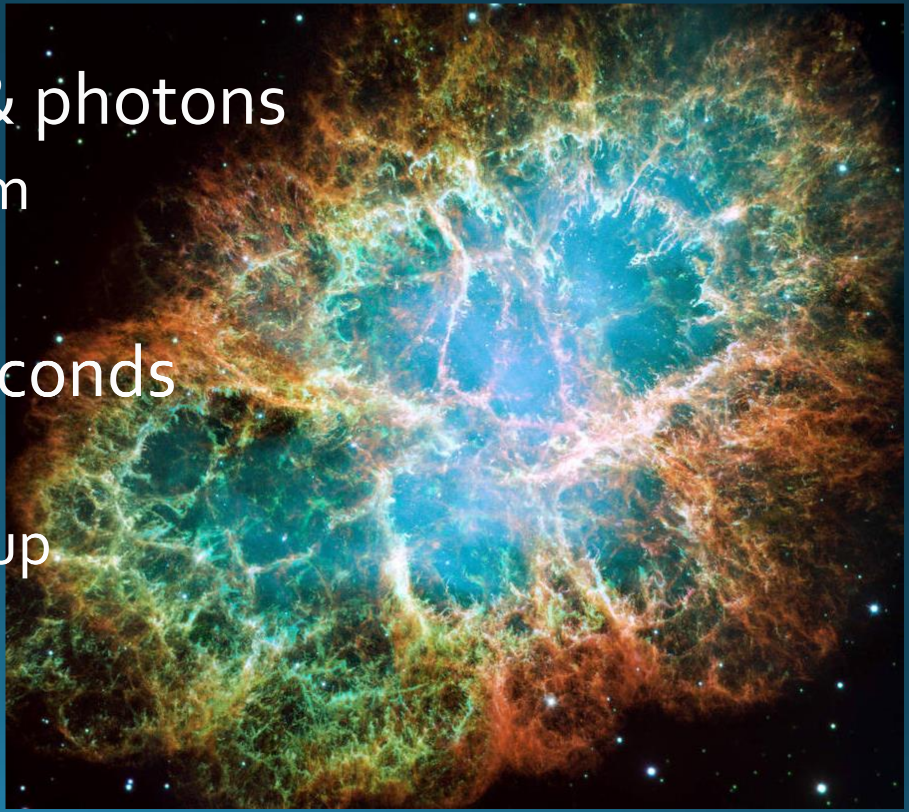
Core-collapse SNe ($M_i > 8M_{sun}$)

- Fe is the most tightly bound nucleus, so now only E source is gravity via contraction
- In core most P contributed by E_F of electrons
  - but T so high that radiation pressure can also be important
- As $E_F / m_e c^2$ goes >> 1, core becomes squashy
  - $dU/dR \sim dU_{grav}/dR \sim R^{-2}$
- As T rises radiation field contains harder photons
  - Some have sufficient E to blast a Fe nucleus apart



Core-collapse SNe

- Electrons have choice: live free or marry a *p* (→ *n*)
  - Cost of living free (E_F) constantly rising → rise in *n/p* ratio (n now stable)
- Result: loss of P contributed by electrons & photons
  - Contraction raises T, exacerbating the problem
  - A runaway; core goes into free-fall
- Myr of nucleosynthesis undone in a few seconds
  - High T and high E_F favour *n*
  - Masses of E required, but gravity can serve it up



Core-collapse SNe

- *e* and γ absorption by nuclei → many free neutrons
- Eventually they become degenerate and start to exert useful pressure
- Collisions become more & more energetic as T and E_F of neutrons rise
  - → copious emission of neutrinos
- The density is so high neutrinos can't simply escape (as from Solar core) but they diffuse out
- Impart momentum to envelope as they brush by
- M of neutron core inevitably close to Chandrasekhar limit
  - if core goes over limit, it collapses to black hole

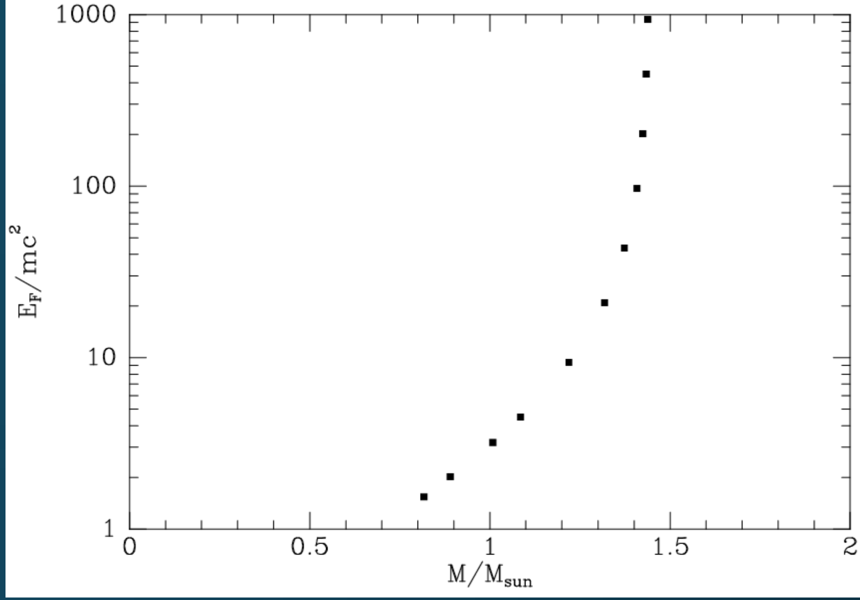
Thermonuclear SNe

- White dwarf = powder keg
  - If heated to ~ 10^{10}K, C+O will release E as they fuse to Ne, Si, etc
  - E released would be enough to unbind star
- Burning in Sun is stable:
  - If T rises, rate of burning increases but also region expands, slowing E production
- In WD reaction rate rises with T, but P depends on E_F rather than T
  - So rise in T increases reaction rate → further rise in T with no regulation until kT reaches E_F
- Consequence: possibility of converting ~M_{sun} to Fe-group elements in seconds

Thermonuclear SNe

- What's the detonator?
- Shock of merging with companion white dwarf?
- M pushed over $M_{Chandra}$ by accretion of gas from binary companion?
- We aren't sure

Chandrasekhar-mass models



- Star becomes squishy
  - Fundamental P mode has very low frequency & large amplitude?
- Rise in central density
  - Gas heats by compression
  - Fusion rate rises steeply with T
- Screening by dense *e* bath allows nuclei to approach very closely
  - Fusion proceeds at low level once $T \sim 10^9$ K
- Core becomes convective
- URCA process (e absorption followed by β decay) cools core by neutrino emission
- System smoulders for ~1000 yr
- Then deflagration front or detonation wave sweeps through converting ~M_{sun} of C+O to Fe-peak elements in ~2 seconds
- E released blasts star into interstellar space

Energetics

$$v_{\rm escape}(M_{\rm C}) = \sqrt{\frac{2GM_{\rm C}}{2000\,{\rm km}}} = 12\,000\,{\rm km\,s^{-1}}$$

$$E(M_{\rm C}) = \frac{GM_\odot^2}{2000\,{\rm km}} = 1.5\times10^{44}\,{\rm J}\ \ (10^{51}\,{\rm erg})$$

- E of SN remnant about this
- E(NS) is 200 times larger because R~10 km
- But E of all SN remnants ~ 10^{44} J because this is E of WD starting point
  - Most E from core-collapse SNe carried off by neutrinos
  - (with help from gravitational waves)

Conclusion

- Stars develop degenerate cores that can explode by two fundamentally different processes
  - Thermonuclear runaway shatters CO WD converting most of it to Fe-group elements
  - Blast of neutrinos following implosion of massive Fe core kicks much of envelope out
- In each case starting point is $M_{Chandra}$ supported by E_F
- KE of debris similar in both cases and ~ binding E of WD
- Big picture is reasonably clear but the details are complex, intractable and so poorly understood